When negative mass replaces both dark matter and dark energy. Solving the problem of primordial antimatter.

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Abstract :

If one tries to include negative masses in the cosmological model one immediately comes up against the runaway effect, violating the action-reaction principle. It is then necessary to opt for a model with two metrics. We take as a starting point Sabine Hossenfelder's model, which is mathematically correct but incompatible with a confrontation with observational data. This approach is taken up again by modifying the field equations whose solutions then make it possible to account for the acceleration of cosmic expansion, the strong gravitational lens effects in the vicinity of galaxies and clusters, and the flatness of the rotation curves. Using the theory of dynamic groups we show that this negative mass is a copy of antimatter, with negative mass and energy. This approach concretizes the idea of Andrei Sakharov and solves the problem of non-observation of primordial antimatter. The remaining questions to be solved to make this model an alternative to the current standard model ΛCDM are listed below.

1 – **What future for cosmology**?

For lack of promising new options, cosmology has for years been largely oriented towards what could be called chimerical worlds. Nowadays, authors submit papers with magic words such as "cosmic strings, monopoles, branes, firewall, quintessence, Chaplying gas, axions", which do not constitute a true model, nor offer any observational confirmation, are much more likely to be published than those that attempt to use classical mathematical and geometric tools that some will consider obsolete or outdated. Nevertheless, it has proved impossible to date to confer an identity to the two components that serve as a basis for the mainstream model ΛCDM, dark matter and black energy. What will happen if, in the coming years, we fail to capture dark matter particles in underground shelters, or in space? What are the alternatives to this model? The Israeli Mordechai Milgrom [1] has suggested considering modified gravity. But

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there is currently nothing to integrate such a concept into Generalized Relativity.Let us add that this idea suffers from observational contradictions. A second alternative is to consider the existence of a matter of negative mass which, emitting negative energy photons, would thus definitively escape optical observations and would only reveal its presence through its gravitational effects

2 – **Two attempts to introduce negative masses into the cosmological model**

These are references [2] and [3]. In the first work the authors try to bring up to date Dirac Milne's cosmological model with equal amounts of positive and negative mass. It is then a cosmos free of gravitational field which follows a law of linear expansion versus time. It is further suggested that this second matter may correspond to primordial antimatter, pushing away our own matter. An advantage put forward is that the cosmological horizon ct then grows in the same way as the universe itself. Under these conditions its homogeneity would be assured whatever the epoch and we would no longer need to resort to the theory of inflation. The authors argue, on the basis of abundance of light elements calculations, that the predictions from such a model would be compatible with observational data. But this model then finds itself in complete contradiction with the ΛCDM model and with the observations reporting the phenomenon of acceleration of expansion ([4], [5], [6]). Finally, attention is drawn to the behavior of antimatter created in the laboratory when it is subjected to the gravitational field of Earth, without explaining why this antimatter would fall upwards. In the second essay [3] the author tries to melt dark matter and dark energy into a single entity of negative mass. To make it play the role attributed to the cosmological constant in the mainstream model ΛCDM it is then necessary that negative mass is continuously created, to maintain its constant density by a phenomenon that is not described in the paper..

(1) $$\rho^{-} = \frac{c^2}{8\pi G}\Lambda = Cst$$

Thus, by pretending to simplify the problem, we are only complicating it. However, this study contains an interesting result based on the idea that galaxies are confined in gaps of negative mass. The author shows that this confinement gives flat rotation curves at the periphery. See figure 1.

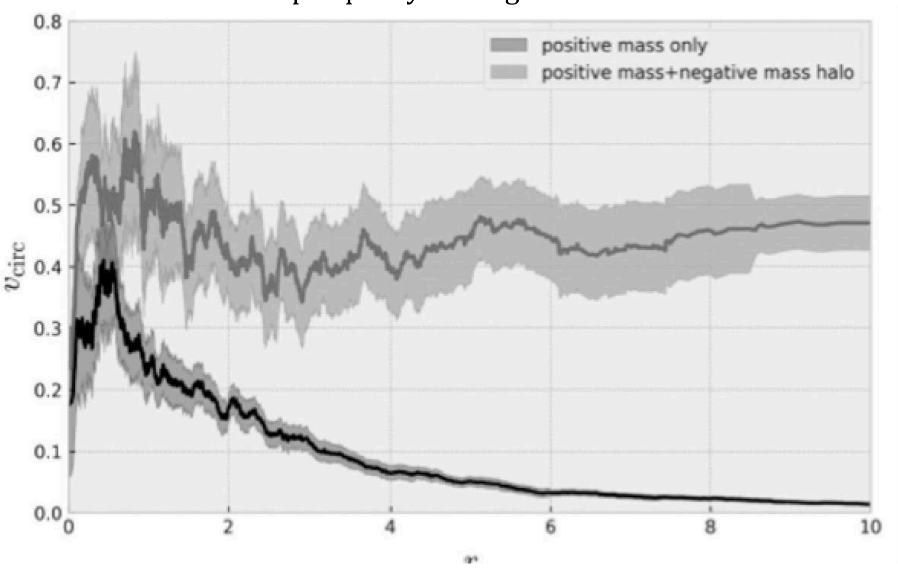


Fig.1 : Circular velocity in a galaxy [3].

But in these two trials the runaway phenomenon remains

3 – **The runaway phenomenon.**

If we consider keeping the current geometric paradigm, we come up against the runaway phenomenon. Indeed Einstein's equation provides the same set of geodesics, referring to test particles, whether they have a positive or negative mass, whatever the source of the field.

(2) $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu}$$

This can be illustrated by examining Schwarzschild's solutions where the mass M, source of the field, can be positive or negative.

(3) $$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

With M>0 we get the figure 2.

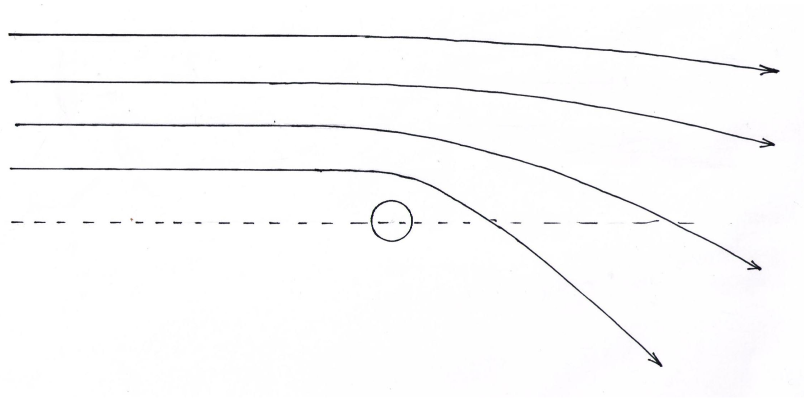


Fig.2 : *Geodesic paths in in viciniy ot a positive mass*

With M<0 we get:

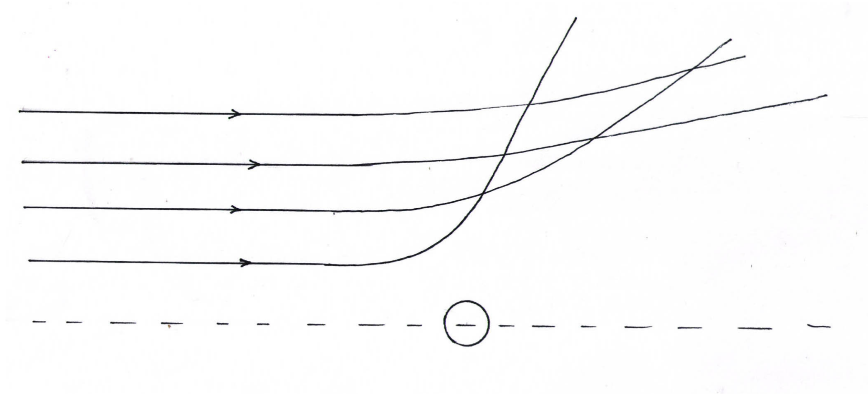


Fig.3 : *Geodesic paths in in viciniy ot a negative mass.*

This can be translated by saying that the positive masses attract everything and the negative masses repel everything. Thus, if we present two masses of opposite signs, the positive mass flees, pursued by the negative mass, which represents a violation of the principle of action-reaction :

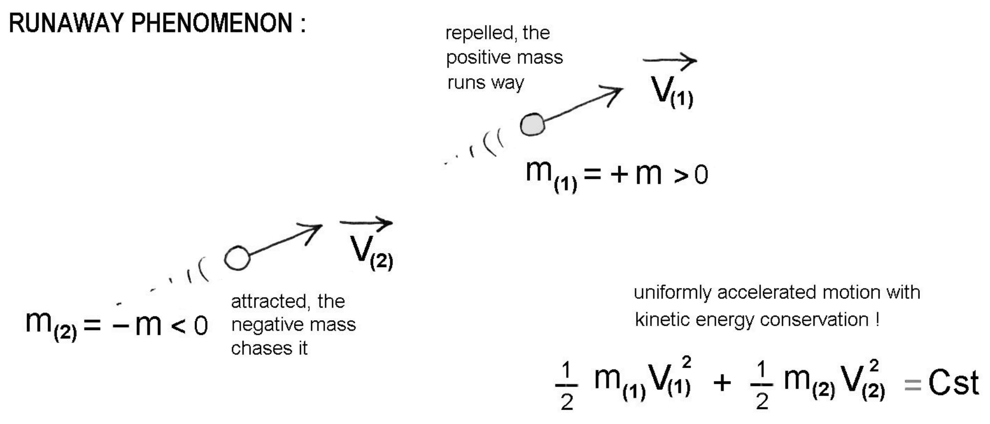


Fig.4 : Runaway effect.

This phenomenon had been highlighted by H. Bondi [7] then by W. Bonnor [8]. In the above-mentioned essays it is suggested that this phenomenon could be at the origin of cosmic rays. If we want to avoid this phenomenon and restore the principles of equivalence and reaction action, we are obliged to consider that masses of opposite signs behave differently in the same field of gravity. It is therefore necessary to consider two geodesic sets, coming from two different metrics

4 – **Massive bigravity.**

In 2002 T.Damour and I.Kogan [9] introduce the formalim of fully non-linear bigravity. They consider two branes, « right » and « left », interacting through massive gravitons. They introduce Lagrangian densities in the action : the Ricci terms $R^R L^R \sqrt{-g^R}$, $R^L \sqrt{-g^L}$, the terms corresponding to positive matter $L^R \sqrt{-g^R}$ and negative matter $L^{(-)} \sqrt{-g^{(-)}}$, are based on the corresponding four-dimensional hypervolumes $\sqrt{-g^R}\ dx^o dx^1 dx^2 dx^3$ and $\sqrt{-g^L}\ dx^o dx^1 dx^2 dx^3$. They introduce an interaction term : $\mu\left(g^R g^L\right)^{1/4} \sqrt{-g^L}\ dx^o dx^1 dx^2 dx^3$. Based on an « average volume factor » $\left(g^R g^L\right)^{1/4}$. The variational method produces a system of two coupled field equations. In the second members are the tensors representing the sources of the two materials "right" and "left" as well as the terms and reflecting the interaction between the two. The authors then consider different models: branes, Kaluza-Klein models, non-commutative geometry. This first article is quickly followed by a second [10]. The scientific community considers at the time that this essay should represent an important contribution since the journal Physical Review D devotes 42 and 56 pages to these two articles, while no data emerged that can be compared with any observational data.

5 – **A bimetric theory with exchange symmetry.**

In 2008 [11] Sabine Hossenfelder published in the journal Physical Review D a theoretical essay entitled "a bimetric theory with exchange symmetry". Both metrics have Lorentzian signatures. The metric **g** is that of ordinary matter. The behavior of the second matter is deduced from a metric $\underline{\mathbf{h}}$. Using what the author calls "pull over", a Lagrangian is proposed:

$$S = \int d^4x \sqrt{-g}\ \left({}^{(g)}R / 8\pi G + L(\Psi) \right) + \sqrt{-h}\ P_{\underline{h}}(\underline{L}(\Phi))$$

(4)

$$+ \int d^4x \sqrt{-\underline{h}}\ \left({}^{(\underline{h})}R / 8\pi G + \underline{L}(\underline{\Phi}) \right) + \sqrt{-\underline{g}}\ P_g(L(\Psi))\ (32)$$

- ${}^{(g)}R$ *and* ${}^{(h)}R$ are Ricci scalars from metrics **g** and $\underline{\mathbf{h}}$.

- g *and* $\underline{h}$ are the determinant of those metrics

- $d^4x\sqrt{-g}$ *and* $d^4x\sqrt{-\underline{h}}$ are the corresponding 4-hypervolumes.

- Ψ et $\underline{\Phi}$ are the matter fields.

The pull over $P_{\underline{h}}$ is an automorphism on the tensor field which maps *h*-fields as the *h*-observer sees them to *h*-fields as the g-observer sees them. Conversely, P_g maps g-fields measured by the g-observers to g-fields as the *h*-observer sees them.

A "double-variation" is operated through a coupling defined in equation (27) of the section **III** [11]:

(5) $$\delta h_{\kappa\lambda} = -\left[a^{-1}\right]^{\mu}_{\kappa}\left[a^{-1}\right]^{\nu}_{\lambda}\delta g_{\mu\nu}$$

This is the covariant version of the coupling relationship we used in our paper [12]. A system of coupled field equations corresponding to equations (34) and (35) of section **IV** of paper [11] is then obtained :

(6) $$^{(g)}R_{\kappa\nu} - \frac{1}{2}g_{\kappa\nu}\,^{(g)}R = T_{\kappa\nu} - \underline{\mathrm{V}}\sqrt{\frac{\underline{h}}{g}}\,a^{\underline{\nu}}_{\nu}\,a^{\underline{\kappa}}_{\kappa}\,\underline{T}_{\underline{\kappa\nu}}$$

$$^{(h)}R_{\underline{\nu\kappa}} - \frac{1}{2}\underline{h}_{\underline{\nu\kappa}}\,^{(\underline{h})}R = \underline{T}_{\underline{\nu\kappa}} - \mathrm{W}\sqrt{\frac{g}{\underline{h}}}\,a^{\nu}_{\underline{\kappa}}\,a^{\kappa}_{\underline{\nu}}\,T_{\nu\kappa}$$

$\underline{\mathrm{V}}$ is the determinant of $P_{\underline{h}}$ and W is the determinant of P_g. The author then studies the solutions of the system without a second member, which allows him, using coupled Schwarzschild solutions, to define the interaction laws, i.e. the inertial masses of the two species. Masses of the same signs attract each other according to Newton's law, while masses of opposite signs repel each other. The principle of action-reaction is thus saved. The system also shows a negative gravitational lens effect that the masses of one population create on the photons of the second population (these being the particles according to the null geodesics of of the metric of the second population). But in the following the author tries to make the evolution in time of the second population follow the evolution of the positive masses by assuming that the density, which involves the gravitational mass, of the second population is positive: $\underline{\rho} > 0$. This is then obtained by arbitrary choices of the signs of the terms present in the Lagrangian. A violation of the principle of equivalence can be deduced from this, pointed out by the author. The aim is, while maintaining a global symmetry between the two systems, to show fluctuations, very non-linear, whose presence could then be equivalent to the addition of a cosmological constant Λ. But the mechanism giving rise to these fluctuations is not described. It is not clear how such fluctuations, occurring in a system where the densities and densities are equal ($\rho \simeq \underline{\rho} > 0$) , could lead to effects of two orders of magnitude higher. By attempting an analogy, it would be like hoping for 50-meter high waves in a basin with 50 cm of water.

6 – **Adaptation of the system of coupled field equations**.

Nevertheless the geometrical and mathematical approach is interesting and mathematically correct. It is the physical interpretation, which translates into the choice of signs in the Lagrangian, which is flawed. We modify these signs of matter to obtain a negative density $\underline{\rho} < 0$ and pressure $\underline{p} < 0$. Two speeds of light c *and* $\underline{c}$, a priori unequal, appear, as well as curvature indices k *and* $\underline{k}$. We introduce two "Einstein constants" χ *and* $\underline{\chi}$, which are also unequal a priori. Using the same mathematical technique, we then obtain the system

$$(7) \qquad {}^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu} \, {}^{(g)}R + \Lambda g_{\kappa\nu} = \chi \left[T_{\kappa\nu} + \underline{\mathrm{V}} \sqrt{\frac{\underline{h}}{g}} \, a^{\underline{\nu}}_{\nu} \, a^{\underline{\kappa}}_{\kappa} \, \underline{T}_{\underline{\kappa}\underline{\nu}} \right]$$

$$(8) \qquad {}^{(h)}R_{\underline{\nu}\underline{\kappa}} - \frac{1}{2} \underline{h}_{\underline{\nu}\underline{\kappa}} \, {}^{(\underline{h})}R + \underline{\Lambda} \, \underline{h}_{\underline{\nu}\underline{\kappa}} = -\underline{\chi} \left[\underline{T}_{\underline{\nu}\underline{\kappa}} + \mathrm{W} \sqrt{\frac{g}{\underline{h}}} \, a^{\kappa}_{\underline{\kappa}} \, a^{\nu}_{\underline{\nu}} \, T_{\nu\kappa} \right]$$

The satisfaction of Bianchi's identities is ensured, by construction, by the application $\boldsymbol{a}$ (reference , equation [13]):

$$(9) \qquad {}^{(g)}\nabla_{\nu} \left[\underline{\mathrm{V}} \sqrt{\frac{\underline{h}}{g}} \, a^{\nu}_{\underline{\nu}} \, a^{\kappa}_{\underline{\kappa}} \, \underline{T}^{\underline{\nu}\underline{\kappa}} \right] = 0$$

$$(10) \qquad {}^{(h)}\nabla_{\underline{\nu}} \left[\mathrm{W} \sqrt{\frac{g}{\underline{h}}} \, a^{\underline{\kappa}}_{\kappa} \, a^{\nu}_{\underline{\nu}} \, T^{\nu\kappa} \right] = 0$$

About the equations without a second member, nothing is changed. But the mass creating these trajectories, as well as the pressure inside it and its density are perceived as negative ($\underline{M} < 0$, $\underline{p} < 0$, $\underline{\rho} < 0$) by an observer made of positive mass. To build the evolution of the two systems in time, two FLRW solutions (metrics that Friedmann-Lemaître-Robertson-Walker) are introduced. In these, speeds c and $\underline{c}$ for photons travelling along geodesics of zero length from the metrics **g** and **h** are introduced.

$$(11) \qquad ds^2 = -c^2 dt^2 + a^2 \left[\frac{dr^2}{1 - kr^2} + d\theta^2 + \sin^2\theta \, d\varphi^2 \right]$$

$$(12) \qquad d\underline{s}^2 = -\underline{c}^2 dt^2 + b^2 \left[\frac{dr^2}{1 - \underline{k}r^2} + d\theta^2 + \sin^2\theta \, d\varphi^2 \right]$$

So we have a unique M_4 manifold equipped with two metrics **g** and **h** . The evolutions of the two systems will be read in the construction of the two scale factors $a(t)$ *and* $b(t)$. Let's write the equations in their mixed form :

(13) $$ {}^{(g)}R^{\nu}_{\kappa} - \frac{1}{2}\delta^{\nu}_{\kappa}\,{}^{(g)}R = \chi\left[T^{\nu}_{\kappa} + \underline{V}\sqrt{\frac{\underline{h}}{g}}\, a^{\nu}_{\underline{\nu}}\, a^{\underline{\kappa}}_{\kappa} \underline{T}^{\underline{\nu}}_{\underline{\kappa}} \right] $$

(14) $$ {}^{(h)}R^{\underline{\kappa}}_{\underline{\nu}} - \frac{1}{2}\delta^{\underline{\kappa}}_{\underline{\nu}}\,{}^{(h)}R = -\underline{\chi}\left[\underline{T}^{\underline{\kappa}}_{\underline{\nu}} + W\sqrt{\frac{g}{\underline{h}}}\, a^{\underline{\kappa}}_{\kappa}\, a^{\nu}_{\underline{\nu}} T^{\kappa}_{\nu} \right] $$

The source tensors are :

(15) $$ T^{\nu}_{\mu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \qquad \underline{T}^{\underline{\nu}}_{\underline{\mu}} = \begin{pmatrix} \underline{\rho}\,\underline{c}^2 & 0 & 0 & 0 \\ 0 & -\underline{p} & 0 & 0 \\ 0 & 0 & -\underline{p} & 0 \\ 0 & 0 & 0 & -\underline{p} \end{pmatrix} $$

By introducing these metric solutions into the two equations we obtain two pairs of differential equations involving the first and second derivatives of the scale factors:

(16) $$ a = \frac{da}{dt} \qquad \ddot{a} = \frac{d^2a}{dt^2} \qquad \dot{b} = \frac{db}{dt} \qquad \ddot{b} = \frac{d^2b}{dt^2} $$

These equations are:

(17) $$ -\chi\left[\rho c^2 + \underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2 \right] = -\Lambda + \frac{3k}{a^2} + \frac{3\dot{a}^2}{c^2a^2} $$

(18) $$ -\chi\left[\rho c^2 + \underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2 + p + \underline{V}\frac{b^3}{a^3}\underline{p} \right] = \Lambda - \frac{k}{a^2} - \frac{\dot{a}^2}{c^2a^2} - \frac{2\ddot{a}}{c^2a} $$

(19) $$ \underline{\chi}\left[\underline{\rho}\,\underline{c}^2 + W\frac{a^3}{b^3}\rho c^2 \right] = -\underline{\Lambda} + \frac{3\underline{k}}{\underline{b}^2} + \frac{3\dot{\underline{b}}^2}{\underline{c}^2\underline{b}^2} $$

(20) $$ \underline{\chi}\left[\underline{\rho}\,\underline{c}^2 + W\frac{a^3}{b^3}\rho c^2 + \underline{p} + W\frac{a^3}{b^3}p \right] = \underline{\Lambda} - \frac{\underline{k}}{b^2} - \frac{\dot{b}^2}{\underline{c}^2b^2} - \frac{2\ddot{b}}{\underline{c}^2b} $$

One cannot, as S. Hossenfelder does, consider only equations (17) and (19) and deduce from them laws of evolution by considering the terms $\underline{V}$ *and* W as independent quantities, when this is not the case. The treatment of the system of the four equations results in equations of compatibility (existence of the solution). This calculation is detailed in Appendix **I**. It is similar to what is obtained by introducing an FLRW metric into Einstein's equation, which leads to the condition

(21) $$ \rho c^2 \mathrm{R}^{3(1+\frac{\alpha}{3})} = Cst $$

In the case of a dust universe this becomes $\rho c^2 \mathrm{R}^3 = Cst$, which is nothing but the translation of energy conservation. Same thing in the case of a universe of radiation where one has : $\rho_r c^2 \mathrm{R}^4 = Cst$.

Equations (17) and (18) give:

(22)
$$\frac{\frac{d}{dt}\left[\rho c^2 + \underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2\right]}{\left[\rho c^2 + \underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2 + p + \underline{V}\frac{b^3}{a^3}\underline{p}\right]} + \frac{3}{a}\frac{da}{dt} = 0$$

And equations (19) and (20) lead to:

(23)
$$\frac{\frac{d}{dt}\left[\underline{\rho}\,\underline{c}^2 + W\frac{a^3}{b^3}\rho c^2\right]}{\left[\underline{\rho}\,\underline{c}^2 + W\frac{a^3}{b^3}\rho c^2 + \underline{p} + W\frac{a^3}{b^3}p\right]} + \frac{3}{b}\frac{db}{dt} = 0$$

For a dust universe:

(24)
$$\frac{d}{dt}\left[\rho c^2 a^3 + \underline{V}\,\underline{\rho}\,\underline{c}^2 b^3\right] = \frac{d}{dt}\left[W\rho c^2 a^3 + \underline{\rho}\,\underline{c}^2 b^3\right] = 0$$

This results in a generalized law of energy conservation, which requires that:

(25)
$$\underline{V} = W = 1$$

(26)
$$\rho c^2 a^3 + \underline{\rho}\,\underline{c}^2 b^3 = E = Cst$$

In a radiation dominated universe:

(27)
$$\underline{V} = \frac{b}{a} = \frac{1}{W}$$

withe the generalized energy conservation equation :

(28)
$$\rho c^2 a^4 + \underline{\rho}\,\underline{c}^2 b^4 = E = Cst$$

If we opt for relation (35) we would then have to rewrite the equations according to:

(29) $$ {}^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu} \, {}^{(g)}R + \Lambda g_{\kappa\nu} = \chi \left[T_{\kappa\nu} + \left(1+\alpha \sqrt[3]{\frac{h}{g}}\right) \sqrt{\frac{h}{g}} \, a_{\nu}^{\underline{\nu}} \, a_{\kappa}^{\underline{\kappa}} \, \underline{T}_{\underline{\kappa\nu}} \right] $$

(30) $$ {}^{(h)}R_{\underline{\nu\kappa}} - \frac{1}{2} \underline{h}_{\underline{\nu\kappa}} \, {}^{(\underline{h})}R + \underline{\Lambda} \, \underline{h}_{\underline{\nu\kappa}} = -\underline{\chi} \left[\underline{T}_{\underline{\nu\kappa}} + \left(1+\alpha \sqrt[3]{\frac{g}{\underline{h}}}\right) \sqrt{\frac{g}{\underline{h}}} \, a_{\kappa}^{\underline{\kappa}} \, a_{\underline{\nu}}^{\nu} \, T_{\nu\kappa} \right] $$

$\alpha = 0$ corresponds to the matter dominated era while $\alpha = 1$ corresponds to the radiation dominated one.

By passing we can show taht :

(31) $$ \chi = -\frac{8\pi G}{c^4} $$

For the second system :

(32) $$ \underline{\chi} = -\frac{8\pi \underline{G}}{\underline{c}^4} $$

By opting for null cosmological constants Λ and $\underline{\Lambda}$, and for matter dominated eras we get

(33) $$ \dot{a} = c \sqrt{-k + \frac{8\pi G}{3c^4} \frac{E}{a}} $$

(34) $$ \dot{b} = \underline{c} \sqrt{-\underline{k} + \frac{8\pi \underline{G}}{3\underline{c}^4} \frac{E}{b}} $$

(35) $$ \frac{\ddot{a}}{a} = -\frac{8\pi G}{3c^2} E $$

(36) $$ \frac{\ddot{b}}{b} = \frac{8\pi \underline{G}}{3\underline{c}^2} E $$

Since the measurements for the matter-dominated era show an acceleration, by interpreting this phenomenon using this model and not by adding a cosmological constant, we deduce that the energy E is negative. This confirms the hypothesis that had prevailed to lead to interesting numerical simulations. Equation (33) imposes the value -1 for the curvature index. Consequently, all negative masses are in a state of deceleration. These equations are therefore written:

(37) $$ \frac{1}{a} \frac{d^2 a}{dt^2} = -\frac{8\pi G}{3c^2} E $$

(38) $$ \frac{1}{b} \frac{d^2 b}{dt^2} = \frac{8\pi \underline{G}}{3\underline{c}^2} E $$

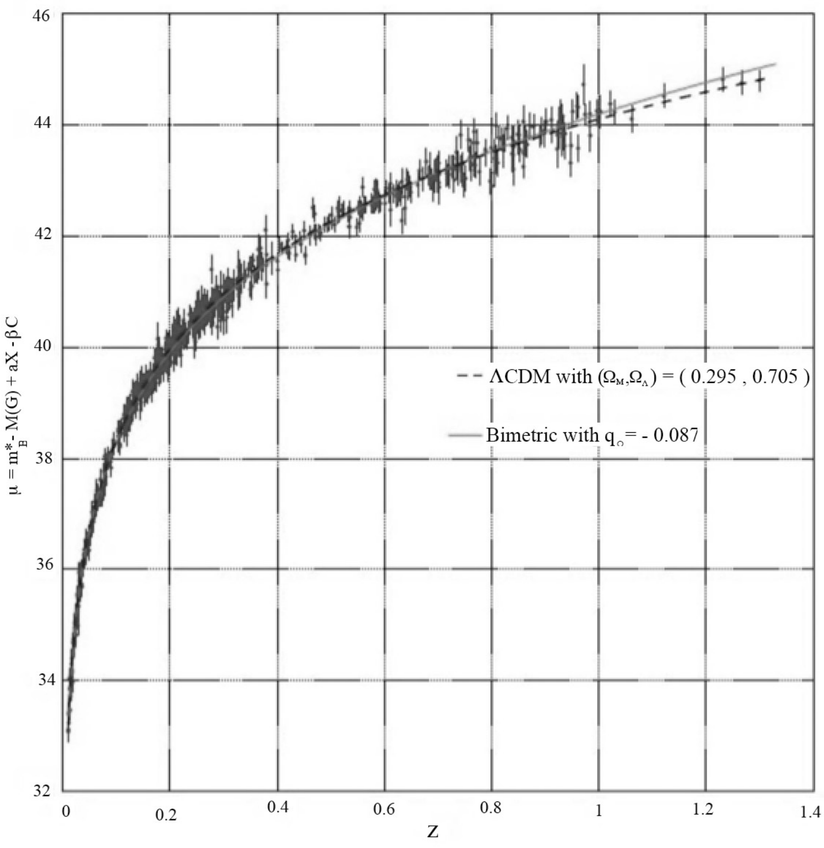
7 – **A universe dominated by negative mass.**

Equation (37) becomes:

$$a^2 \frac{d^2 a}{dt^2} = \frac{8\pi G}{c^2} \left| E_o \right| a_o^3 \tag{39}$$

The exact solution of this equation was given by W. Bonnor [8]. Only the portion of this curve corresponding to the matter phase, should be considered. Indeed, when the curve is extended towards the origin of time, a non-zero value of the scale factor *a* is obtained. In its portion referring to the material phase the model then accounts for the acceleration of the expansion ([4],[5],[6]). The mainstream model ΛCDM predicts an exponential acceleration that results from the constancy of the negative energy density associated with the cosmological constant Λ. In the proposed model the negative energy is that of negative mass. It decreases as it expands. The evolution of the scale factor of positive mass (39) then tends towards an asymptote, reflecting a linear expansion as a function of time..

The magnitude values as a function of distance, corresponding to this model, were compared with data from 700 type Ia supernovae with excellent agreement [14]



Hubble diagram compared with the two models (linear redshift scale)

Fig.5 : Comparison of the model with data from 700 supernovae Ia [14]

At this stage the interest of the approach is that it melts the two invisible components of the universe into a single entity, the negative mass.

8 – **The radiation dominated era.**

If we assume that this era can still be described by FLRW metrics we get $\underline{V} = \frac{1}{W} = \frac{b}{a}$. If in this era the dynamic is always dominated by the content of the universe in negative energy we would always have a positive acceleration in the sector of positive masses :

$$a^3 \ddot{a} = \frac{8\pi G}{3c^4} \left| E_o \right| a_o^4 \tag{40}$$

At this stage no explanation is given for the very important asymmetry between the two cosmic entities. In the next article the origin of this asymmetry will be explained and modelled, starting from a totally symmetrical initial situation. This extension of the model will explain in passing the extreme homogeneity of the early universe, without using the inflation model, and will provide an alternative interpretation of the fluctuations of the CMB, the cosmological radiation background.

Let us now go back to the description provided by the model of its material-dominated era and see what results from it.

9 – **With respect to local observations**.

The agreement is immediate. As S. Hossenfelder pointed out in section IV of her article:

> *« Since both kinds of matter repel, one would expect the amount of h-matter in our vicinity to presently be very small ».*

Thus the system becomes :

$$^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu} \, ^{(g)}R + \Lambda g_{\kappa\nu} = \chi T_{\kappa\nu} \tag{41}$$

$$^{(\underline{h})}R_{\underline{\nu\kappa}} - \frac{1}{2} \underline{h}_{\underline{\nu\kappa}} \, ^{(\underline{h})}R + \underline{\Lambda} \, \underline{h}_{\underline{\nu\kappa}} = - \underline{\chi} \sqrt{\frac{g}{\underline{h}}} \, a_{\kappa}^{\underline{\kappa}} \, a_{\underline{\nu}}^{\nu} \, T_{\nu\kappa} \tag{42}$$

The first equation is then identified with Einstein's equation. Thus the model accounts for local observations such as the advance of Mercury's perihelion and the deviation of light rays by the mass of the Sun.

10 - **The structure on a very large scale. Simulation results** [15].

As early as 1995 based on the simple hypothesis concerning the laws of interaction, namely:

- Masses with same sign mutually attract through Newton's law. .
- Masses with opposite signes mutuelly repel throught « anti-Newton's law »

Numerical simulations were performed, adding the hypothesis of the pre-eminence of negative mass content ($\left|\underline{\rho}\right| >> \rho$). The Jeans times of the two species then differ:

$$t_J^{(-)} = \frac{1}{\sqrt{4\pi\, G\left|\rho^{(-)}\right|}} << t_J^{(+)} = \frac{1}{\sqrt{4\pi\, G\,\rho^{(+)}}} \qquad (43)$$

It is therefore the negative mass that first forms a regular series of conglomerates, confining the positive mass in the residual space, thus giving it a lacunar, alveolar structure:

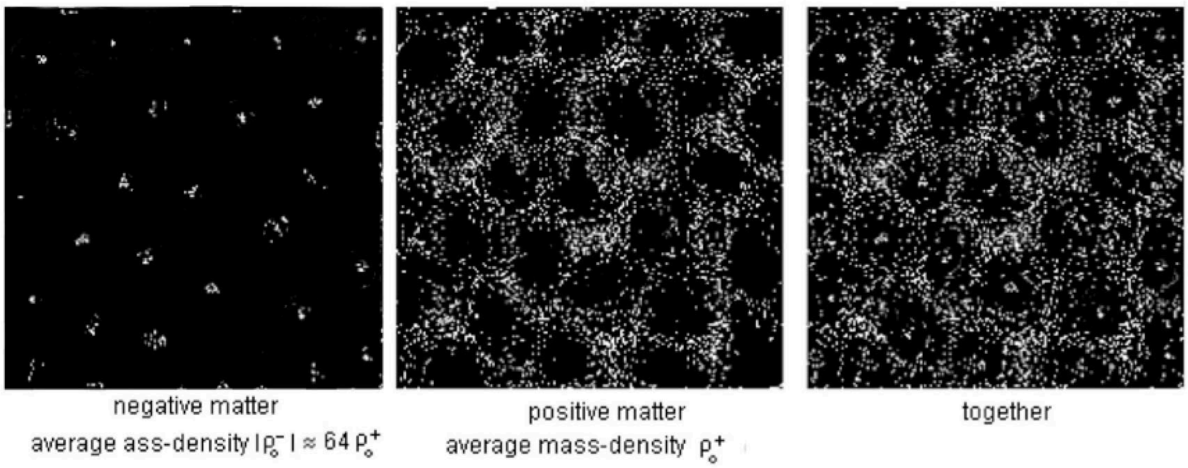


Fig.6 : Result of 1995 simulations [15] with $\left|\rho^-\right| >> \rho^+$

It would be very interesting to be able to resume this simulation work, in 3D with the current computers, to which we unfortunately do not have access. This diagram, where positive mass material is distributed in the manner of joined soap bubbles, causes filaments to form at the junction of three cells, the clusters at the junction of four. This pattern also leads to a different mode of galaxy formation. When this very large-scale structure is formed, immediately after decoupling, the negative mass conglomerates exert a significant counter-pressure on the matter, which is compressed along plates. The result is a sudden heating. But the flat structure allows a dissipation of this thermal energy by radiation, which destabilizes the environment and serves as a trigger for the birth of galaxies.

11) **The Great Repeller.**

In 2017 [16] a 3D cartography of the universe has been published, located in a cube of one and a half billion light years on the side, containing hundreds of thousands of galaxies. At this scale the speed of expansion of the galaxies reaches 20,000 km/s. The authors were able to determine the proper motions of these galaxies by subtracting from the velocity measurements the velocity corresponding to Hubble's law. It was already known for a long time that there was a region of space, called the Great Attractor, towards which the galaxies seemed to converge. But the analysis revealed the existence, in the diametrically opposite direction, 600 million light-years away from our galaxy, of an empty region that seemed to push galaxies in all directions. This totally empty portion of space was given the name Great Repeller..

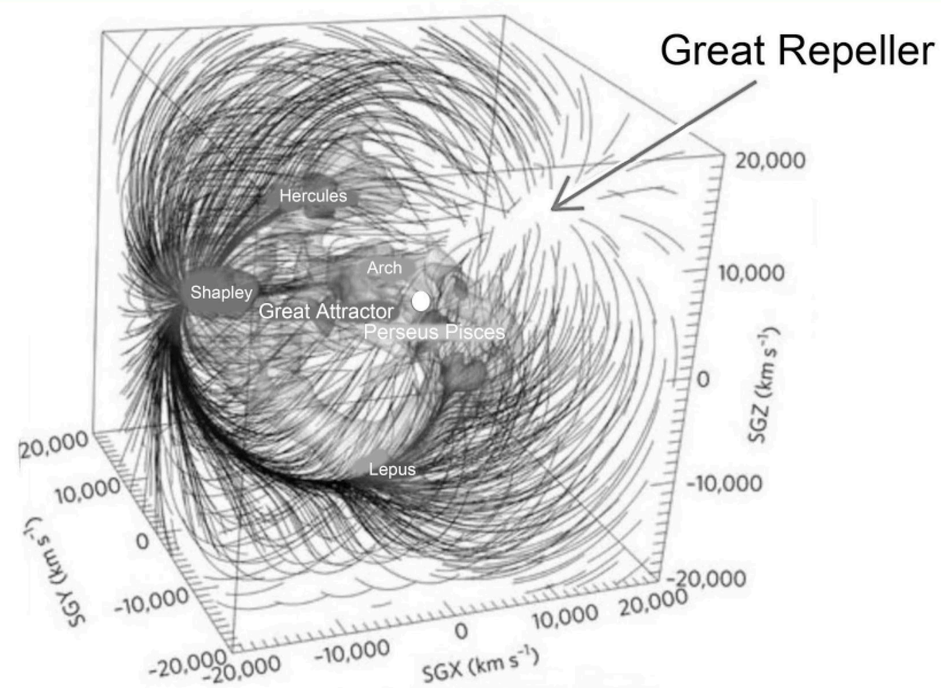


Fig.7 : The so-called Great Repeller.

Proponents of the hypothesis of the existence of dark matter, of positive mass, have imputed such a phenomenon as being due to a vast gap in the general distribution of this medium of unknown nature. But gravitational instability generates conglomerates of matter, not gaps. The alternative interpretation resulting from the present model attributes this phenomenon to the presence of a conglomerate of negative mass, invisible because it emits photons of negative energy. These objects decrease the luminosity of distant sources by negative gravitational lens effect and, in fact, galaxies with high redshift ($z > 7$) are classically considered as dwarfs. We think that they are similar to nearby galaxies but that it is this effect that makes them appear as dwarfs. Future advances in observational means should allow us to highlight a marked contrast in magnitude at the center of the formation, allowing us to evaluate the diameter of the conglomerate. If such a contrast, very localized, is found, it will be necessary to abandon the interpretation by a gap in the dark matter. From this angle the model is thus falsifiable. At this stage the model is an alternative to the ΛCDM model with the same percentage of visible matter

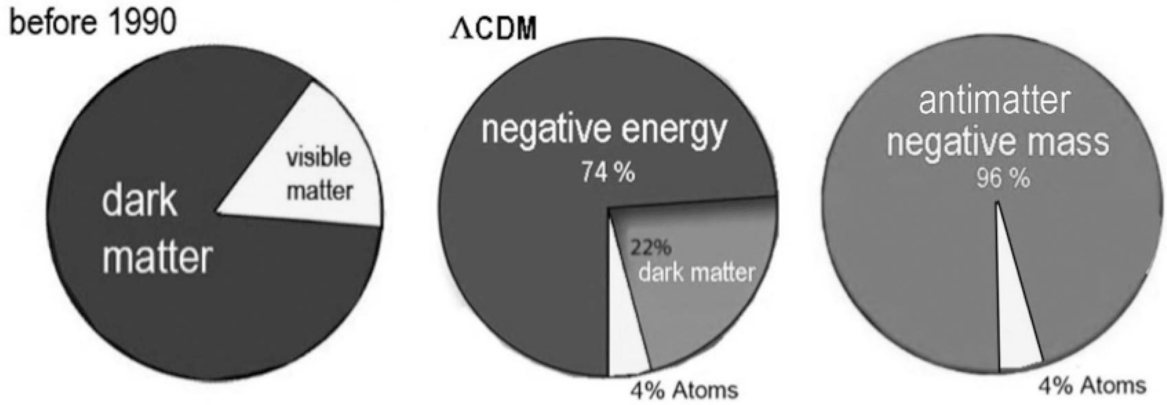


Fig.8 : The components.

12 – **Confirmations at the level of galaxies and galaxy clusters**.

As shown by the simulations [15] built from the interaction laws and the hypothesis of the dominance of the negative mass, the two populations are mutually exclusive. When galaxies form, negative mass immediately invades the space between them. These galaxies then occupy gaps in the distribution of negative mass which, exerting a counter-pressure on them, ensures their confinement. This confinement force makes possible high rotational speeds at the periphery. Confer the curve of figure 1. These gaps in the negative mass are equivalent to the action of a positive mass, images of these gaps with an inverted mass sign. They thus generate strong gravitational lens effects, in the vicinity of galaxies and clusters, and endow these clusters with escape velocities that exceed the value calculated using only the visible mass of the cluster.

13 - **Construction of a galaxy model. Spiral structure**.

Galaxies are self-gravitating, non-collisional mass point systems, which therefore fall within the Vlasov equation.

(44)
$$\frac{\partial f}{\partial t} + \vec{v} \,.\, \vec{\nabla}_r f - \vec{\nabla}_r \Psi . \vec{\nabla}_v f = 0$$

Ψ is the gravitational potential and ρ mass density, coupled through the Poisson equation :

(45)
$$\Delta \Psi = 4\pi G \rho$$

Such an approach was sketched in 1942 by S. Chandrasekhar [17], limited to Maxwell-Boltzmann type velocity distribution functions. $\vec{v}_o = <\vec{v}>$ being the macroscopic velocity it is then possible to express the Vlasov equation in terms of residual velocity $\vec{V} = \vec{v} - \vec{v}_o$ (the speed of thermal agitation in fluid mechanics). By introducing [18] the operator

(46) $$D \equiv \frac{\partial}{\partial t} + \vec{V}_o . \vec{\nabla}_r$$

we then consider two Vlasov equations linked by the Poisson equation:

(47) $$Df + \vec{V} . \vec{\nabla}_r f - \vec{\nabla}_v f . \left(\vec{\nabla}_r \Psi + D \vec{v}_o \right) - \vec{\nabla}_v \vec{V} : \vec{\nabla} \vec{v}_o = 0$$

(48) $$\underline{D} \underline{f} + \underline{\vec{V}} . \vec{\nabla}_r \underline{f} - \vec{\nabla}_v \underline{f} . \left(\vec{\nabla}_r \Psi + \underline{D} \underline{\vec{v}}_o \right) - \vec{\nabla}_v \vec{V} : \vec{\nabla} \underline{\vec{v}}_o = 0$$

(49) $$\Delta \Psi = 4 \pi G (\rho + \underline{\rho})$$

The terms $\vec{\nabla}_v \vec{V}$ and $\vec{\nabla} \vec{v}_o$ are dyadic matrices [18] formed from different vectors and the macroscopic velocity gradient. The term $\vec{\nabla}_v \vec{V} : \vec{\nabla} \vec{v}_o$ represents the scalar product of two dyads ([18] page 16 eq. 1.31.4) defined by $\mathbf{A} : \mathbf{B} = A_i^j B_j^i$. The logarithm of the Maxwell-Boltzmann distribution function is a spherical polynomial with respect to the components (U,V,W) of the residual velocity. Elliptic solutions [19] corresponding to an axisymmetric system have been constructed. The spheroid of velocities then gives way to an ellipsoid of velocities, closer to the observational reality (Vertex). The solution (stationary and spherically symmetrical) used corresponds, for positive masses, to:

(50) $$\mathrm{Log}\, f = \mathrm{Log}\, A(r) - \frac{V^2}{< v^2 >} + a(r) (\vec{V} . \vec{r})^2$$

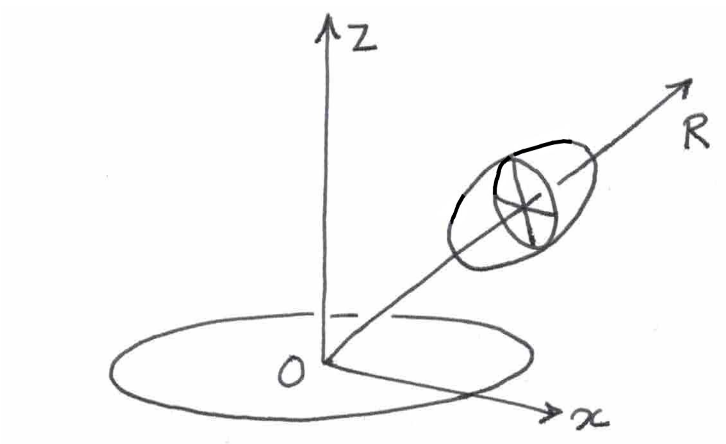


Fig.9 : Velocity ellipsoid in spherically symmetric system.

For negative masses, a Maxwell-Boltzmann function has been chosen:

(51) $$\mathrm{Log}\, \underline{f} = \mathrm{Log}\, \underline{A}(r) - \frac{V^2}{< \underline{v}^2 >}$$

By introducing these distribution functions into equations (47) and (48) and by coupling, in a steady state, to the Poisson equation we obtain systems of differential equations from which we can produce an exact solution thanks to the power of the calculation of the dyadic algebra [18]. The solution corresponds to figure 8 and can represent a spheroidal galaxy, or a cluster of galaxies, confined by its negative mass environment.

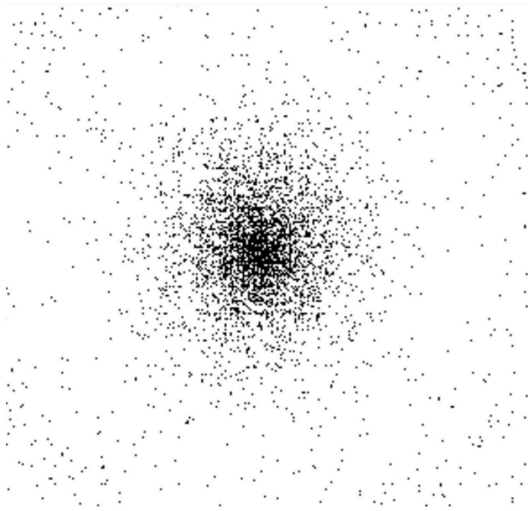


Fig.10 : Spheroidal galaxy, or cluster of galaxies.

A rotational movement is then introduced as a solid body. The image below is extracted from the results of a 2D simulation carried out in 1992 at the DESY laboratory in Hamburg by Frédéric Descamp, then a student. At the end of a few turns constituting a transitory regime, a barred spiral galaxy was formed, lasting for thirty turns, which was then of the order of the age of the universe.

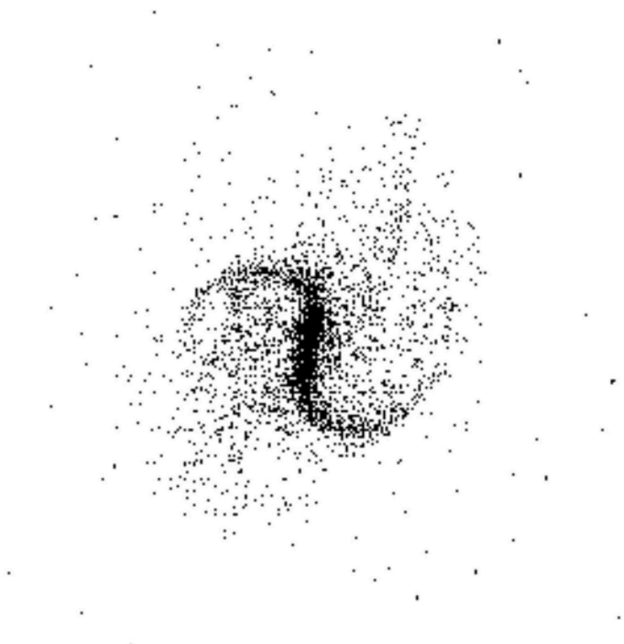


Fig.11 : Barred spiral from 2D simulations (1992 : 2x10,000 points)

The evolution of the kinetic moment reflects a strong dynamic friction effect with a slowing down of the rotational motion of the galaxy. At the same time, the velocity profile is modified.

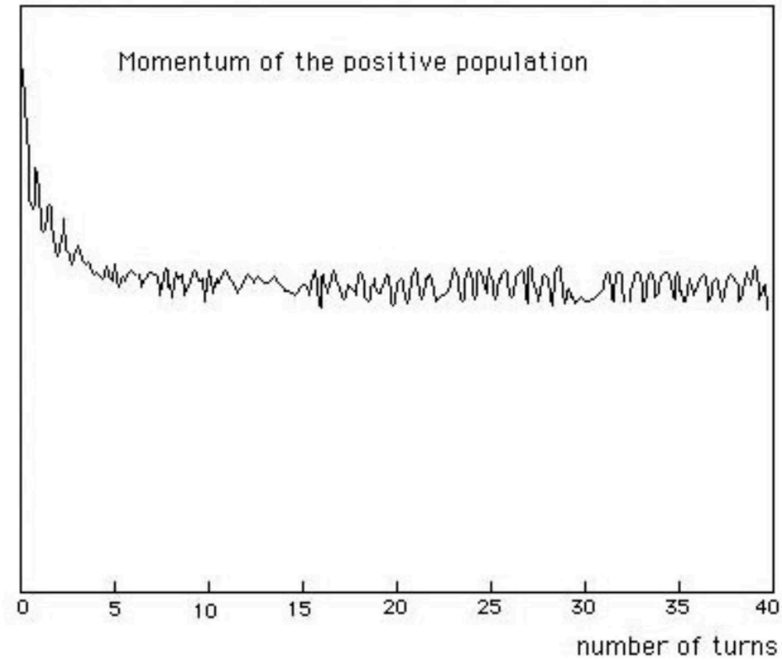


Fig.12 : Kinetic momentum evolution (1992 : 2x10,000 points)

Galaxies are non-collision systems. Therefore the transfer of momentum and energy does not occur through particle encounters. This tells us how non-collisional systems evolve: through the appearance of density waves. In this case the galaxy made of positive masses interacts with its negative mass environment through the density waves that form within it. These waves have their counterpart in the world of negative masses. The fact that spiral structures, artificially introduced as initial conditions in the simulations where only positive masses are present, dissipate is then explained by the fact that these waves have no medium with which to interact. Let us give an analogy. When water flows through the drain of a bathtub, when the water thickness is low, spiral waves appear. They result from the increased incidence of frictional forces on the bottom of the bathtub. If these frictional forces were to disappear, so would the spiral waves, which would then lose their purpose. These waves are analogous to shock waves, because the rotational speed of the fluid then exceeds the speed of propagation of surface waves. It is the same for galaxies where the local value of the rotational velocity exceeds the average value of the residual velocity (equivalent for galaxies, of the thermal agitation velocity in a fluid). Unfortunately, these simulations could only be continued in this laboratory for a few months, as the student was quickly called to order by his hierarchy. At the time, the Marseille observatory had appropriate computing resources, but we were unable to access them. Attempts to publish in all the specialized journals resulted in the answer "sorry, we don't publish speculative works". After a few years we abandoned this direction of research.

14 – **The nature of this negative mass. The primordial antimatter.**

According to the generally accepted pattern, matter and antimatter are formed from radiation. As S. Weinberg wrote in his famous book "the first three minuts", at that time the universe was filled by "different types of radiations", in the sense that the synthesis reactions of the matter-antimatter pair from radiation followed one another at a rapid rate, then their annihilation by giving back the same radiation, in a situation of "detailed balancing". With the expansion the radiation is no longer energetic enough to give birth to the pairs and the annihilations take over. The fact that one pair in a billion survived remains unexplained. In 1967 the Russian Andrei Sakharov suggested [20] the existence of two universes, having in common only the initial singularity, the Big Bang. In one of the universes, ours, the synthesis of matter from quarks would have been slightly faster than that of antimatter from antiquarks. At the time of this great annihilation would have remained this remnant of matter, associated with its equivalent in antiquarks, in a 3/1 ratio, both being drowned in the photons resulting from the annihilations. Inverse situation in this twin universe where one would count a remainder of antimatter associated with its equivalent in quarks, always in a 3/1 ratio. In order to maintain a kind of generalized symmetry A.Sakharov also suggested when in this other universe the arrow of time is reversed (T-symmetry), as well as the orientation of space (P-symmetry). But he had not envisaged an interaction between these two sheets of universes.

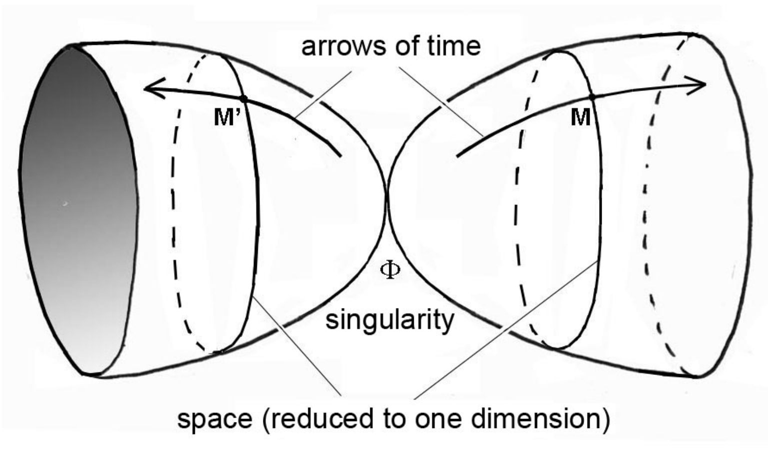


Fig.13 : 2D didactic ime of the Sakharov model.

Topologically, one can envisage a folding of the hypersurface according to a M_4 manifold with two folds.

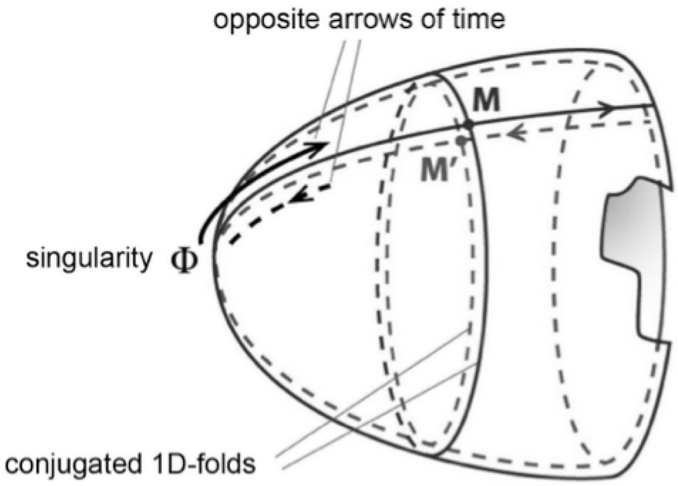


Fig .14 : The Sakharov model, folded. .

By the way, if the Big Bang punctual singularity is replaced by a tubular bridge, such a configuration brings an original answer to the question "what was the Big Bang" (or "what was there before Planck's time")

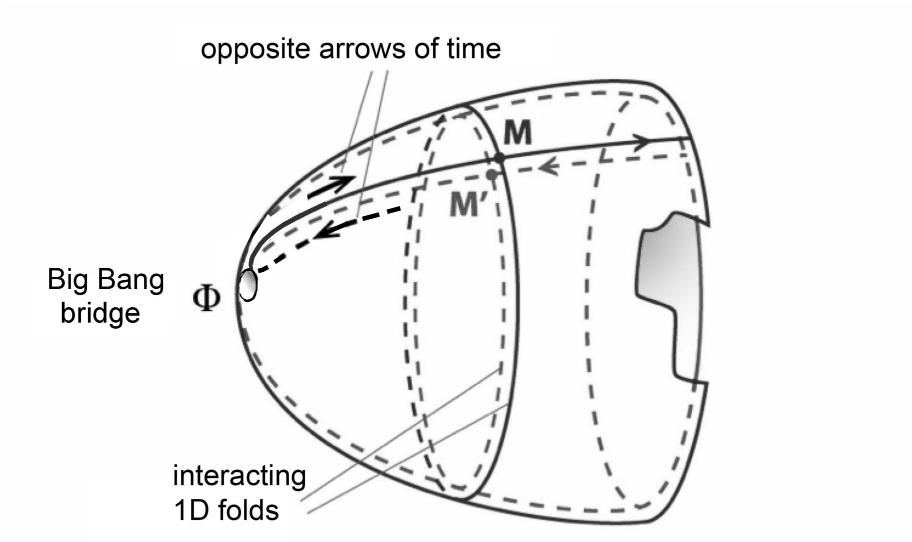


Fig15 : A space bridge replaces the point-like sungularity.

Is there a connection between the model on the previous pages and this question of time reversal? The answer is positive. It should be understood that it is the inversion of the time coordinate, not an inversion of the direction of travel along geodesics, i.e. of the proper time. This question of "T-symmetry" is present in the Quantum Theory of Fields [21] where we find operators T and P for time and space inversions. Apart from these Quantum Mechanics considerations, this possible inversion of space and time coordinates belongs to Lorentz's group, axiomatically defined by:

(52) $$ {}^{T}L\,G\,L = G \quad \text{avec} \quad G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} $$

Thus defined, the complete Lorentz group has four related components:

- L_n : Neutral component, subgroup.. Inverts neither time nor space.
- L_s : Elements reversing space (P-symmetry)
- L_t : Elements reversing time (T-symmetry)
- L_{st} : Elements reversing time and space (PT-symmetry)

The first two sets of elements make up the orthochrone subgroup:

(53) $$ L_o = L_n \cup L_s $$

The other two form the antichroneous subgroup:

(54) $$ L_a = L_t \cup L_{st} $$

Using :

(55) $$ \{-L_n\} \equiv \{L_{st}\} \qquad \{-L_s\} \equiv \{L_t\} $$

v:

(56) $$ L = \lambda L_o \quad \textit{with} \quad \lambda = \pm 1 $$

This makes it possible to define the complete Poincaré group as an extension of the restricted Poincaré group (orthochrone):

(57) $$ \begin{pmatrix} \lambda L_o & C \\ 0 & 1 \end{pmatrix} \quad \textit{with} \quad C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} $$

which is then Minkowski's space isometry group. As the mathematician J.M.Souriau [22] showed in 1970, it is then possible to show the characteristics of the motion of particles as components of the moment, this constituting the theory of dynamic groups. At this stage we classify particles as sets of motions within a Minkowski space. These are characterized by:

- Energy E

- Impulsion **p**

- Spin **s** (not quantified)

If we consider that physics is limited to the declination of possible movements corresponding to the restricted Poincaré group, to the orthochronous subgroup, then Minkowski's space is populated only by orthochronous movements, traversed in a past-future direction, of energy and positive mass.

The various movements are declined by making act on the dual of the Lie algebra of the group, on its space of the moments the relations of the group:

(58)
$$\begin{aligned} M' &= L_o M\, {}^tL_o + C\, {}^tP\, {}^tL_o - L_o P\, {}^tC \\ P' &= L_o P \end{aligned}$$

which are the equations (13.107) of reference [22]. But if we opt for the complete group this becomes:

(59)
$$\begin{aligned} M' &= L_o M\, {}^tL_o + \lambda C\, {}^tP\, {}^tL_o - \lambda L_o P\, {}^tC \\ P' &= \lambda L_o P \end{aligned}$$

($\lambda = -1$) results in PT-symmetry. We then observe that the antichronal elements, which then inscribe in Minkowski's space movements resulting in the inversion of time, also inverts the energy E and the pulse **p**, which make up the quadrivector P (reference [22], equation (16.67)).:

(60)
$$P = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

and, beyond that, due to the fact that E = mc^2, inverts the mass. This represents a first step in the identification of the present model with that of A.Sakharov. To go further, one must refer to the geometrical definition of antimatter symmetry [23]. Space-time must then be given an additional dimension ζ (Kaluza space). Starting from the Poincaré orthochronous group we have the first extension of the group :

(61)
$$\begin{pmatrix} \mu & 0 & \phi \\ 0 & L_o & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} \quad with \quad \xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad and \quad \mu = \pm 1$$

The movements of the particles take place in a 5-dimensional space $\left\{ \zeta , t , x , y , z \right\}$. Along this fifth dimension a simple translation $\zeta \rightarrow \zeta + \phi$ takes place. These translations thus constitute a one-parameter subgroup. Noether's theorem will thus make that this subgroup will be associated with the conservation of a scalar which will be the electric charge q . By calculating the action of the group, an additional equation will appear

$$\begin{aligned} & q' = \mu q \\ (62) \qquad & M' = L_o M\, {}^t L_o + C\, {}^t P'\, {}^t L_o - L_o P'\, {}^t C \\ & P' = L_o P \end{aligned}$$

The terms ($\mu = -1$) thus duplicate the movements of matter by reversing the direction of travel of the fifth coordinate, as well as the electric charge q. But the latter is only one of the p quantum charges q_i. We can then consider an extension of Minkowski's space according to p additional dimensions:

$$\left\{ \zeta^1 , \zeta^2 , \ldots , \zeta^p , t , x , y , z \right\}$$

through the group :

$$(63) \qquad \begin{pmatrix} \mu & 0 & \ldots & 0 & 0 & \phi_1 \\ 0 & \mu & \ldots & 0 & 0 & \phi_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mu & 0 & \phi_p \\ 0 & 0 & \ldots & 0 & L_o & C \\ 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \ldots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix}$$

The p translations according to the p additional dimensions go hand in hand with p quantum numbers to which we can assign the values $\left\{ -1 , 0 , +1 \right\}$. Particles of matter have quantum charges null or equal to +1. The symmetry ($\mu = -1$) leads to the inversion of all quantum charges, i.e. a C-symmetry. This inversion of the direction of travel of all the additional dimensions ζ_i thus represents the geometrical interpretation of matter/antimatter symmetry [23]. Among these sets of motions there are some for which all quantum charges are zero and which correspond to positive energy photons.

In Sakharov's idea, the two sheets of universe, disjointed at home, combined in the present model, translated a CPT symmetry. One can translate this [25] by the group:

$$(64) \qquad \begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad with \quad \begin{aligned} \lambda &= \pm 1 \\ \mu &= \pm 1 \end{aligned}$$

The coadjoint action of the group on the dual of its Lie algebra gives then:

$$q_i' = \lambda \mu q_i$$

(65) $$M' = L M {}^tL + C {}^tP {}^tL - L P {}^tC = L_o M {}^tL_o + \lambda C {}^tP {}^tL_o - \lambda L_o P {}^tC$$

$$P' = L P = \lambda L_o P$$

($\lambda = -1$) is then translated by a CPT-symmetry. Orthochronous and antichronous movements correspond to movements, either of positive energy and mass, or of negative energy and mass of the same particles: neutrons, protons, electrons. Within these populations the duality of matter/antimatter is present.

There are therefore two types of antimatter:

- -An antimatter of positive energy and mass, C-symmetrical of the matter that we will call antimatter in the sense of Dirac.

- An antimatter of negative energy and mass, PT-symmetrical to our ordinary matter, which we call antimatter in the sense of Feynmann.

We thus give substance to Andrei Sakharov's idea. If the rates of production of matter and antimatter in the two entities were different, this determines the nature of negative mass matter. It is therefore:

- antiprotons
- antineutrons
- anti-electrons

of negative energy and mass, mixed with photons of negative energy and a remnant, in a 3/1 ratio, of negative energy quarks. These particles, emitting negative energy photons, are not detected by our optical instruments.

$$q_i' = \lambda \mu q_i$$

(66) $$M' = L M {}^tL + C {}^tP {}^tL - L P {}^tC = L_o M {}^tL_o + \lambda C {}^tP {}^tL_o - \lambda L_o P {}^tC$$

$$P' = L P = \lambda L_o P$$

For details of the share calculation, see Annex **II**.

15 – **Astrophysics of negative masses.**

We have not, at this stage, provided a description of the evolution in the radiative era. We can suppose that in this negative world a primordial nucleosynthesis will take place, giving rise to anti-helium and anti-lithium of negative mass. But after decoupling these atoms are formed spheroidal conglomerates, comparable to immense proto-stars. The cooling time increasing with the mass of the objects, that of these protostars exceeds the age of the universe. These objects slowly lose energy by emitting negative energy photons in the red and infrared range, like protostars. But this negative world will not generate any galaxies or stars and will not be able to synthesize heavy atoms. No planet will be able to form and life will be absent.

16 – **Opening of a new research field in Quantum Mechanics** [24]**.**

It is well known that the equations of relativistic quantum mechanics (Klein-Gordon, Dirac) naturally highlight negative energy states. They have always been eliminated by considering that they lead to negative probability densities. The solution that physicists have found is then to replace, in a rather artificial way it must be admitted, the so-called probability densities by charge densities: this is the birth of antiparticles (in the commonly accepted sense).

However, if we look a little closer at these probability densities, we see that they can be reinterpreted as real probabilities, positive, if we consider that negative energy states are also associated with negative masses. This is particularly striking with the Klein-Gordon density, which involves the ratio

(67) $$\frac{\mathrm{E}}{\mathrm{m}}$$

Probabilistic interpretation is therefore compatible with negative energy states provided that energies and mass are simultaneously negative. And how could it have been otherwise with Einstein's relation at rest E =m c^2 ? Thus quantum mechanics is the ideal ground for reintegrating negative energies. However, the consequences must be discerned. One of them is that the temporal reversal operator will henceforth be considered as a linear and unitary operator [24].

We know, in fact, that a symmetry operator must necessarily be :

- Linear and unitary (LU)

- Anti-linear and anti-unitary (AA)

It is customary to choose, for a spatial inversion P, the choice LU and for a temporal inversion T the choice AA. Thus, the action of these discrete symmetries on the fundamental operators of quantum mechanics as well as on the imaginary i can be summarized by:

(68a) $$\mathrm{P}: \vec{x} \to -\vec{x} \;,\; \vec{\mathrm{p}} \equiv -i\hbar\,\vec{\nabla} \to -\vec{\mathrm{p}} \;,\; i \to i$$

(68b) $$\mathrm{T}: \vec{x} \to \vec{x} \;,\; \vec{\mathrm{p}} \equiv -i\hbar\,\vec{\nabla} \to -\vec{\mathrm{p}} \;,\; i \to -i$$

The fundamental relationship of quantum mechanics:

(69) $$\left[x_j \,,\, \mathrm{p}_k\right] = i\,\hbar\,\delta_{jk}$$

is then invariant under (68a) as well as under (68b).

Moreover, the symmetry $\mathrm{PT}(\,t \to -t\,, i \to -i\,)$ thus chosen ensures the invariance of the energies

(70) $$\mathrm{E} \rightarrow \mathrm{H} \equiv i\hbar \frac{\partial}{\partial t}$$

Positive energies, if we limit ourselves to them, therefore remain exclusively positive. On the contrary, in [24] we have opted for the choice LU, for the two inversions, which leads to the following result:

(71a) $$\mathrm{P}: \vec{x} \rightarrow -\vec{x} \ , \ \vec{\mathrm{p}} \equiv -i\hbar \vec{\nabla} \ \rightarrow \ -\vec{\mathrm{p}} \ , \ i \ \rightarrow \ i$$

(71b) $$\mathrm{T}: \vec{x} \rightarrow \ \vec{x} \ , \ \vec{\mathrm{p}} \equiv -i\hbar \vec{\nabla} \ \rightarrow \vec{\mathrm{p}} \ , \ i \ \rightarrow \ i$$

both ensuring the invariance of (67). The major difference is that the symmetry PT leads this time to a change of sign at the level of the energies:

(72) $$\mathrm{H} \equiv i\hbar \frac{\partial}{\partial t} \ \rightarrow \ -\mathrm{H}$$

There is nothing to prevent, physically, mathematically and from a probabilistic point of view, to consider these negative energy states as long as they are assigned a negative mass. Moreover, it even seems that this additional possibility is more rigorous if we stick to mathematics, since implying that T is linear, it is in agreement with its usual realization (cf. [28] Eq (2.3.16), p. 58, for example):

(73) $$\mathrm{T} = diag(-1,1,1,1)$$

17 – **Conclusion**

This work suggests a complete upheaval of cosmology, as well as an extension of theoretical physics to the field of negative energies. It is a real paradigm shift. While authors will easily be able to place in journals articles that can be qualified as chimerical, containing a certain number of buzzwords, without these leading to any model and any agreement with observation, if we do not want this article to be immediately rejected, simply because its validity will seem so unlikely to the editor of the journal that he will refuse to submit it to a referee, it is necessary hereafter to recall the rigor of its construction and the mass of observational confirmations that accompany it:

- We have shown that the introduction of negative masses in the cosmological model was only possible by switching to a bimetric system corresponding to a system of two coupled field equations. This was the only way to escape the runaway effect and to restore the principles of equivalence and action-reaction. In the physical extension of a previous mathematically coherent work by S. Hossenfelder [11], we have constructed a model where the Newtonian approximation leads to the following interaction scheme:

The masses of the same signs attract each other according to Newton's law.

The masses of opposite signs repel each other according to "anti-Newton".

- The fact that negative masses, of negative energy, emit photons of negative energy, which our instruments cannot capture, explains why certain contents of the universe escape observation.

- Using the theory of dynamic groups, we have shown that in this second matter, of negative energy, matter-antimatter symmetry was also present. It is suggested that this one was simply composed of the same antimatter elements, with negative masses and energies.

- Taking up the ideas formulated in 1967 by Andrei Sakharov, according to which the synthesis of the antimatter of antiquarks would have been faster than the synthesis of the matter of quarks, in this second population, we concluded that these invisible elements did not correspond to dark matter of positive mass, but to antimatter of negative mass: antiprotons, antineutrons, antielectrons, mixed with s photons of negative energy and a residue of quarks of negative energy, which resolves the paradox of the non-observation of primordial antimatter.

- We then constructed an exact solution, based on the hypothesis (which will be justified in a later article) of the prevalence of negative masses. This solution gives a very good agreement with data from 700 type Ia supernovae. The acceleration of the cosmic expansion is thus explained, without using the cosmological constant Λ .

- Since negative mass has negative energy, it takes the place of this unidentified dark energy. It is not vacuum energy and it simply corresponds to a contribution of negative energy quantum states.

- We have shown that these negative energy states emerge naturally from Dirac's equation. An approach that opens up major perspectives in Quantum Mechanics.

- Considering that the negative mass content is negligible in the vicinity of the Sun, the conclusion is that the model is in agreement with local relativistic observations: Mercury perihelion advance, deviation of light rays by the mass of the Sun.

- Still according to the hypothesis of the dominance of a negative mass content, we have shown that the latter is the first to form a set of spheroidal conglomerates which, by pushing matter back into interstitial space, give it a lacunar structure.

- This pattern of large-scale structure formation suggests a new pattern of galaxy formation. The positive mass, compressed into plates, and thus heated, can rapidly dissipate this energy in the form of radiation and thus present a favorable configuration for the birth of galaxies.

- We have suggested that the recently discovered Great Repeller (2017) could betray the presence of a negative mass conglomerate.

- Such objects would cause a reduction in the magnitude of the objects in the background. We think that this is the real reason for the low magnitude of galaxies at $z > 7$.

- We say that future progress in measurements referring to distant objects should make it possible to determine the diameter of such a conglomerate by highlighting locally an important contrast in magnitude.

- We suggest that these conglomerates, composed of anti-hydrogen and anti-helium of negative mass, would behave like immense proto-stars with a cooling time greater than the age of the universe. Thus, the fusion could not start. These objects would produce neither galaxies, nor stars, nor planets, nor atoms heavier than those of which they are made.

- Life would be absent in this negative world.

- On a smaller scale, the negative gravitational lens effect due to a negative mass environment, which ensures the confinement of galaxies, explains the importance of the gravitational lens effects measured in their vicinity.

- Same thing to explain, in clusters, the agitation velocities of galaxies, higher than their liberation velocity calculated on the basis of the mass of the cluster, deduced from the observations. The negative mass environment is again responsible for this confinement effect.

- This configuration also explains the flat shape, at the periphery, of the rotation curves of the galaxies.

- We provide a model of a spheroidal galaxy, built on the basis of two Vlasov equations coupled by the Poisson equation. This elliptical solution has a "vertex"

- Using this model as a support for numerical simulations, it was possible to model the birth of a barred spiral galaxy, which lasts for thirty revolutions.

- This explains why simulations carried out by other authors had led to the rapid dissipation of the spiral arms, artificially introduced under the initial conditions. These structures represent the way in which dissipative processes take place in these systems without collision, reflecting an exchange of energy and momentum through density waves present in both systems. If there is no second system to interact with, the spiral waves disappear.

- In addition, it was stated at the end of the paper that the description of the radiative age and the explanation of the establishment of this strong asymmetry would be present in a future paper.

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Appendix I

Compatibility conditions

The equations are:

(a)
$$-\chi\left[\rho c^2 + \underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2\right] = -\Lambda + \frac{3k}{a^2} + \frac{3\dot{a}^2}{c^2a^2}$$

(b)
$$-\chi\left[p + \underline{V}\frac{b^3}{a^3}\underline{p}\right] = \Lambda - \frac{k}{a^2} - \frac{\dot{a}^2}{c^2a^2} - \frac{2\ddot{a}}{c^2a}$$

(c)
$$\underline{\chi}\left[\underline{\rho}\,\underline{c}^2 + W\frac{a^3}{b^3}\rho c^2\right] = -\underline{\Lambda} + \frac{3\underline{k}}{\underline{b}^2} + \frac{3\dot{\underline{b}}^2}{\underline{c}^2\underline{b}^2}$$

(d)
$$\chi\left[\underline{p} + W\frac{a^3}{b^3}p\right] = \underline{\Lambda} - \frac{\underline{k}}{b^2} - \frac{\dot{b}^2}{\underline{c}^2b^2} - \frac{2\ddot{b}}{\underline{c}^2b}$$

A linear combination of (a) and (b) gives :

(e) $$-\frac{\chi}{2}\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2+p+\underline{V}\frac{b^3}{a^3}\underline{p}\right]=\Lambda-\frac{3\ddot{a}}{c^2 a}$$

Anothe one :

(f) $$-\frac{\chi}{2}\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2+p+\underline{V}\frac{b^3}{a^3}\underline{p}\right]=\frac{k}{a^2}+\frac{\dot{a}^2}{c^2a^2}-\frac{2\ddot{a}}{c^2a}=\frac{k}{a^2}-\frac{1}{c^2}\frac{d}{d\zeta}\left(\frac{\dot{a}}{a}\right)$$

We differentiate (a) with respect to t :

(g) $$-\chi\frac{d}{d\zeta}\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2\right]=-\frac{6k}{a^3}\dot{a}+3\frac{d}{dt}\left(\frac{\dot{a}^2}{c^2a^2}\right)=-\frac{6k}{a^3}\dot{a}+6\frac{\dot{a}}{a}\frac{d}{dt}\left(\frac{\dot{a}}{c^2a}\right)$$

combining to (f) :

(g) $$\frac{d}{d\zeta}\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2\right]+3\frac{\dot{a}}{a}\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2+p+\underline{V}\frac{b^3}{a^3}\underline{p}\right]=0$$

ou

(h) $$\frac{d\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2\right]}{\left[\rho c^2+\underline{V}\frac{b^3}{a^3}\underline{\rho}\,\underline{c}^2+p+\underline{V}\frac{b^3}{a^3}\underline{p}\right]}+3\frac{da}{a}=0$$

En traitant de la même manière les equations (27) et (28) on obtient :

(i) $$\frac{d\left[\underline{\rho}\,\underline{c}^2+W\frac{a^3}{b^3}\rho c\right]}{\left[\underline{\rho}\,\underline{c}^2+W\frac{a^3}{b^3}\rho c+\underline{p}+W\frac{a^3}{b^3}p\right]}+3\frac{db}{b}=0$$

Appendix II

Calculation of the group's action on its space of moments.

The group is represented by the matrices:

(a)
$$a = \begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad with \quad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

For convenience of calculation we will carry out this one with

(b)
$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & L & C \\ 0 & 0 & 1 \end{pmatrix} \quad with \quad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

The element of its Lie algebra is then:

(b)
$$Z \equiv \begin{pmatrix} 0 & 0 & \varepsilon \\ 0 & \delta L & \gamma \\ 0 & 0 & 0 \end{pmatrix}$$

The group is differentiated in the vicinity of its neutral element. Under these conditions δL can be put in the form $\mathrm{G}\,\omega$ where G is the Gramm matrix and ω an antisymmetric matrix

(c)
$$\mathrm{Z} = \begin{pmatrix} 0 & 0 & \varepsilon \\ 0 & \mathrm{G}\,\omega & \gamma \\ 0 & 0 & 0 \end{pmatrix}$$

For computational convenience, we write the action of the group on its Lie algebra $Z' = a^{-1} Z\, a$ instead of $Z' = a\, Z\, a^{-1}$, which is equivalent to computing the action of the inverse of the element of the group on the element of its Lie algebra, but the result will be equivalent since the set of inverses also represents the group. It comes :

(d)
$$\begin{pmatrix} 0 & 0 & \varepsilon' \\ 0 & G\omega' & \gamma' \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \lambda\mu\varepsilon \\ 0 & G^t L\omega L & \gamma G^t L G + G^t L\omega C \\ 0 & 0 & 0 \end{pmatrix}$$

which gives :

(e)
$$\varepsilon' = \lambda\mu\varepsilon$$

$$\omega' = {}^t L\omega L$$

$$\gamma' = G^t L G\,\gamma + G^t L\omega\, C$$

We are looking for the dual of the group's action on its Lie algebra. The element of this Lie algebra depends on 11 parameters.

(f) $$Z=\left\{ \omega_{sx}, \omega_{sy}, \omega_{sz}, \omega_{fx}, \omega_{fy}, \omega_{fz}, \gamma_t, \gamma_x, \gamma_y, \gamma_z, \varepsilon \right\}$$

The moment space of the group will thus be a vector space of dimension 11. It can be put in the form of an antisymmetric matrix M of format (4,4), depending on six parameters, a quadrivector P and a scalar q. The duality can thus be ensured by the constancy of the scalar:

(g) $$\frac{1}{2}\mathrm{Tr}\,(\mathrm{M}\,\omega) + {}^t\mathrm{P}\mathrm{G}\,\gamma + \mathrm{q}\,\varepsilon$$

Ce qui donne :

(h) $$\frac{1}{2}\mathrm{Tr}\,(\mathrm{M}\,\omega) + {}^t\mathrm{P}\mathrm{G}\,\gamma + \mathrm{q}\varepsilon = \frac{1}{2}\mathrm{Tr}\,(\mathrm{M}'\,{}^t\mathrm{L}\omega\,\mathrm{L}) + {}^t\mathrm{P}'\mathrm{G}\,(\mathrm{G}\,{}^t\mathrm{L}\omega\mathrm{C} + \mathrm{G}\,{}^t\mathrm{L}\mathrm{G}\gamma) + \mathrm{q}'\lambda\mu\varepsilon$$

It comes immediately:

(i) $$q = \lambda\mu q'$$

(j) $${}^t\mathrm{P} = {}^t\mathrm{P}'\,{}^t\mathrm{L} \quad \rightarrow \quad \mathrm{P} = \mathrm{L}\,\mathrm{P}'$$

We know that we can perform a circular permutation in the trace:

(k) $$\mathrm{Tr}(\mathrm{M}'\,{}^t\mathrm{L}\omega\,\mathrm{L}) = \mathrm{Tr}(\mathrm{L}\,\mathrm{M}'\,{}^t\mathrm{L}\omega)$$

The identification on the ω terms gives

(l) $$\frac{1}{2}\mathrm{Tr}(\mathrm{M}\omega) = \frac{1}{2}\mathrm{Tr}(\mathrm{L}\mathrm{M}'\,{}^t\mathrm{L}\omega) + {}^t\mathrm{P}\,{}^t\mathrm{L}\omega\mathrm{C}$$

The term ${}^t\mathrm{P}\,{}^t\mathrm{L}\omega\mathrm{C}$ is the scalar product of the row vector ${}^t\mathrm{P}$ by the column vector ${}^t\mathrm{L}\omega\mathrm{C}$. We can therefore write, after having performed a circular permutation in the trace

(j) $${}^t\mathrm{P}\,{}^t\mathrm{L}\omega\mathrm{C} = \mathrm{Tr}(\,{}^t\mathrm{L}\omega\,\mathrm{C}\,{}^t\mathrm{P}) = \mathrm{Tr}(\,\mathrm{C}\,{}^t\mathrm{P}\,{}^t\mathrm{L}\omega\,)$$

By making a circular permutation in the trace. Thus the equation (l) provides:

(j) $$\mathrm{M} = \mathrm{L}\mathrm{M}'\,{}^t\mathrm{L} + 2\mathrm{C}\,{}^t\mathrm{P}'\,{}^t\mathrm{L}$$

But

(k) $$\mathrm{C}\,{}^t\mathrm{P}\,{}^t\mathrm{L} = \frac{1}{2}\left[\mathrm{sym}(\mathrm{C}\,{}^t\mathrm{P}\,{}^t\mathrm{L}) + \mathrm{antisym}\left(\mathrm{C}\,{}^t\mathrm{P}\,{}^t\mathrm{L}\right)\right]$$

Knowing that the trace of the product of a symmetrical matrix by an antisymmetrical matrix is equal to zero:

(l) $$\mathrm{Tr}\left[(\mathrm{C\,PT\,{}^tL} + \mathrm{L\,P\,{}^tC})\times\omega\ \right] = 0$$

It remains:

(m) $$\frac{1}{2}\mathrm{Tr}(\mathrm{M}\omega) = \frac{1}{2}\mathrm{Tr}(\ \mathrm{L\,M'\,{}^tL}\omega) + \frac{1}{2}\mathrm{Tr}\left[(\mathrm{C\,{}^tP\,{}^tL} - \mathrm{L\,P^t\,C})\times\omega\right]$$

Which provides the last equation of the group's action on its moment:

(n) $$\mathrm{M} = \mathrm{L\,M'\,{}^tL} + \mathrm{C\,P'\,{}^tL} - \mathrm{L\,P'\,{}^tC}$$

We make the inversion parameter reappear by $L = \lambda\, L_o$ and group the results together

(o) $$\mathrm{q} = \lambda\mu\,\mathrm{q'}$$

(p) $$\mathrm{M} = \mathrm{L_o\,M'\,{}^tL_o} + \lambda\mathrm{C\,P'^T L_o} - \lambda\mathrm{L_o\,P'\,{}^tC}$$

(r) $$\mathrm{P'} = \lambda\mathrm{L_o\,P}$$

P is the energy-impulsions 4-vector :

(s) $$\mathrm{P} = \begin{pmatrix} \mathrm{E} \\ \mathrm{p}_x \\ \mathrm{p}_y \\ \mathrm{p}_z \end{pmatrix}$$

Equations (o),(p),(r) represent an extension of equations 13.107 of reference [27]. The relation (r) makes it possible to find Souriau's relation ([27] page 190, equations 14.67). The inversion of time ($\lambda = -1$) leads to the inversion of energy and of the impulse vector $\vec{\mathrm{p}}$. The matrix M depending on six parameters can be decomposed into two vectors. The vector f is what Souriau calls the pass and s is the spin.

(t) $$\mathrm{M} = \begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix}$$

The passage f is not an intrinsic attribute of the motion because it can be cancelled by a change of variable accompanying the particle. Only the spin remains, of which Souriau demonstrated in 1970 its geometrical nature. By cancelling the spatio-temporal translation C the relation (p), where λ does not appear, shows that the inversion of time does not modify the spin vector. With this way of carrying out the calculation one obtains the result of the action of the group on a movement, characterized by the quantities $\left\{ E', \vec{p}', \vec{s}' \right\}$ gives another movement $\left\{ E, \vec{p}, \vec{s} \right\}$. It is the relation (o) which informs on the fact that starting from a motion representing that of a particle of matter :

- ($\lambda = -1 ; \mu = 1$) results in a PT-symmetry plus a C-symmetry . One thus obtains the movement of a particle of negative mass.

- ($\lambda = 1 ; \mu = -1$) operates a C-symmetry. The movement obtained is that of an antiparticle in the sense of Dirac, of positive mass.

- ($\lambda - = 1 ; \mu = -1$) represents a PT-symmetry. The motion is that of an antiparticle of negative mass (antiparticle in the sense of Feynmann).